Phonon deficit effect and solid state refrigerators based on superconducting tunnel junctions.

G.G. Melkonyan[a], H. Kröger[a] and A.M. Gulian[b]
[a] *Département de Physique Université Laval, Québec, Québec G1K 7P4, Canada*
[b] *Naval Research Laboratory, Washington DC 20375, USA*

Since the first demonstration of electron cooling in superconductor–insulator–normal metal (SIN) tunnel junctions, there is growing interest in tunneling effects for the purpose to develop on–chip refrigerators which can generate cooling in the order of $100mK$ in an environment of 0.3–0.5K. Thin film devices have the advantage of being extremely compact, operate in a continuous mode, dissipate little power, and can easily be integrated in cryogenic detectors. Motivated by such possibilities, we investigate the phonon deficit effect in thin film SIS (superconductor–insulator–superconductor) and SIN tunnel junctions. Under certain circumstances, the phonon absorption spectra of such tunnel junctions have spectral windows of phonon absorption/emission. We propose to use phonon filters to select the phonon absorbtion windows and thus to enhance the cooling effect. Membranes attached to such tunnel junctions can be cooled in this way more effectively. We discuss a particular superlattice design of corresponding phonon filters.

I. INTRODUCTION

Solid state micro–refrigerators ($SSMR$) have many advantages over conventional cooling devices: they have a long life-time, they are compact and simple to apply. These $SSMRs$ utilize motion of electrons rather than atoms and molecules. The flow of electric current through a semiconductor–semiconductor contact is accompanied by a transfer of heat (the Peltier effect) from one semiconductor to the other. Micro-refrigerators based on the Peltier effect are working at temperatures above $150K$. Solid-state micro-refrigerators operating below $150K$ are a scientific and technical challenge. A possible approach to this problem is to consider electron kinetics in superconductor–insulator–normal metal (SIN) or superconductor–insulator–superconductor (SIS) tunnel junctions [6, 15]. As we will demonstrate in this article, another even deeper understanding of the same effects is possible from the concept of the phonon deficit effect (PDE). The phonon deficit effect was initially predicted [9, 10] to occur when superconductivity is enhanced in high frequency electromagnetic fields (UHF) [4]. Later the PDE was recognized to exist in more general cases [11, 12]. It is related to the violation of detailed balance of kinetic processes in superconductors in the presence of an external field. If a superconducting film is immersed into a heatbath and the electron system goes over to an excited nonequilibrium state due to an external supply of energy, then under appropriate conditions the superconducting film absorbs phonons from the heatbath rather than emitting phonons.

The situation is essentially similar in the case of thin tunnel junctions of superconductors. Phonon emission of a symmetric SIS tunnel junction has been considered in detail in Ref. [11] and an asymmetric junction has been considered in Refs. [8, 12, 18]. In the sub–threshold tunneling regime ($V < 2\Delta/e$, where Δ denotes the gap) the resulting phonon flux is shown in Fig.[1].

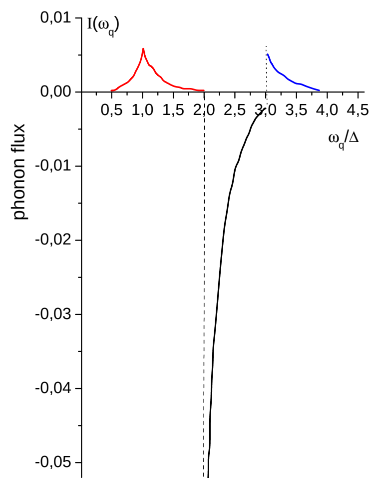

FIG. 1: Spectral dependence of the phonon flux (in units of energy per unit time) radiated from a thin superconducting film (with smaller gap Δ^S) in an SIS' tunnel junction (taken from Ref. [12])

One observes the phonon emission in the frequency windows $0 < \omega_q < 2\Delta/\hbar$ and $\omega_q > 3\Delta/\hbar$. In the same time there is the phonon absorption in the frequency window $2\Delta/\hbar < \omega_q < 3\Delta/\hbar$. The analogy between this case and the superconductor in the UHF field [see Ref. [12], Figure 6.2] strikingly demonstrates the universality of the mechanism of the phonon deficit effect.

Experiments on refrigeration are usually performed on the SIN tunnel junctions. So called "cooling by extraction" is recognized as the operational principle of these type of refrigerators. At temperatures much below the critical temperature T_c of the superconducting electrode (S) there are few thermally excited quasiparticles above the superconducting gap Δ, so that tunneling from S to

the normal metal electrode (N) can be neglected. Meanwhile, in the normal metal electrode (N) there are enough electron-like and hole-like thermal excitations near the Fermi energy E_F which can tunnel to S and effectively modify the electron distribution in N. The rate of electronic heat exchange with the lattice is proportional to $T_e^5 - T_{ph}^5$, where T_e is electronic temperature and T_{ph} is the temperature of phonons. The cooling process is a balance between phonon heat flow, quasiparticle extraction and Joule heating. The value of the applied potential V on the tunnel junction determines if cooling or heating occurs depending on which process is dominant in the heat transfer balance.

Here we have studied the behavior of SIN tunnel junction using the phonon deficit effect. The outline of this paper is as follows. In Section II we describe the phonon heatbath model and decoupling of electrons from phonons. In Section III we solve kinetic equations for the SIN tunnel junction. We calculate the net phonon flux to the SIN tunnel junction when electron and phonon systems are maintened at different temperatures. We show that the rule for heat exchange $P \propto T_e^5 - T_{ph}^5$ follows from the PDE in SIN tunnel junctions. In analyzing the PDE we propose the phonon filters. We present the results of our numerical simulations, which can be utilised to enhance the refrigeration effect. The constraction of phonon filters in accordance with the calculated phonon absorbtion spectra is discussed in Section IV. Conclusions are given in Section V.

II. PHONON HEATBATH MODEL AND DECOUPLING OF ELECTRONS FROM PHONONS

Experimentally junction electrodes are thin films placed on a substrate. These thin films should be considered as a system of two interacting subsystems: electrons and phonons. Nonequilibrium dynamics of phonons and electrons in superconductors is described by a coupled system of kinetic equations [12]. In the simplest case, when the lattice phonons play the role of a thermostat, the kinetic equations for electron (hole) excitations and phonons in a metal can be written as in the following form [12]

$$\frac{\partial n_{\pm\epsilon}}{\partial t} = Q^f\left(n_{\pm\epsilon}\right) + I^{el-ph}\left(n_{\pm\epsilon}, N^i_{\omega_q}\right), \tag{1}$$

$$\frac{\partial N^i_{\omega_q}}{\partial t} = L\left(N^i_{\omega_q}\right) + I^{ph-el}\left(N^i_{\omega_q}, n_{\pm\epsilon}\right) \ . \tag{2}$$

Here $n_{\pm\epsilon}$ denotes the distribution functions of nonequilibrium electron-like (n_ϵ), respectively hole-like ($n_{-\epsilon}$) excitations in the film. $N^i_{\omega_g}$ represents the distribution function of internal phonons, $I^{el-ph}(n_{\pm\epsilon}, N^i_{\omega_q})$ stands for the electron–phonon collision integral, and $I^{ph-el}(N^i_{\omega_q}, n_{\pm\epsilon})$ denotes the phonon–electron collision integral. $Q^f\left(n_{\pm\epsilon}\right)$ is an external source of nonequilibrium quasiparticles and f represents external fields to be specified later. $L(N^i_{\omega_q})$ is an operator describing the interaction of phonons with an external heatbath.

An important problem in a dynamical theory of metals is the study of nonequilibrium stationary states, i.e. when $\partial n_{\pm\epsilon}/\partial t = 0$, $\partial N^i_{\omega_q}/\partial t = 0$. For thin films in an external field the dynamical problem can be simplified by considering the phonon heatbath model [5, 12] and also the "geometric-acoustical" approximation of phonon propagation. In this approximation the distribution function of internal phonons is given by $N^i_{\omega_q} = N^0_{\omega_q}$, where $N^0_{\omega_q}$ is the distribution function of phonons at thermodynamic equilibrium. The distribution of electrons and holes corresponds to a nonequilibrium state. In a steady state, using the phonon heatbath model, the following system of equations holds

$$-\,Q^f\left(n_{\pm\epsilon}\right) = I^{el-ph}\left(n_{\pm\epsilon}, N^0_{\omega_q}\right), \tag{3}$$

$$-L\left(N^0_{\omega_q}\right) = I^{ph-el}\left(N^0_{\omega_q}, n_{\pm\epsilon}\right) \ . \tag{4}$$

These equations are uncoupled. Eq.(3) can be used to obtain stationary steady state solutions for the distribution functions of nonequilibrium excitations (electron and hole branches. Eq.(4) becomes an identity, which describes the phonon exchange (phonon flux) between the thin film and its environment [12].

III. SIN TUNNEL JUNCTION - KINETIC EQUATIONS IN THE STEADY STATE

We consider now a massive superconductor (injector S'), having a superconducting gap Δ', joined with a thermostat held at temperature T. A thin film having a thickness $d \sim \xi_0$ (superconductor or normal metal) is attached to the massive superconductor by means of a thin oxide layer. It has the same temperature (see Fig.[4]). In the dissipative steady state, nonequilibrium electron (n_ϵ) and hole ($n_{-\epsilon}$) excitations in the thin film (S or N) are described by the following kinetic equation (see Eqs. (1) and (3))

$$u_\epsilon \frac{\partial n_{\pm\epsilon}}{\partial t} = Q(n_{\pm\epsilon}) + I^{el-ph}(n_{\pm\epsilon}, N^0_\omega) = 0 \ . \tag{5}$$

Here $u_\epsilon = |\epsilon|\theta(\epsilon^2 - \Delta^2)/\sqrt{\epsilon^2 - \Delta^2}$ denotes the BCS density of states, ϵ is the quasiparticle energy and $Q(n_{\pm\epsilon}) \equiv Q^f\left(n_{\pm\epsilon}\right)$ describes the injector. In a particular geometry the external field f represents the distribution function of quasiparticles in the electrode denoted by S'. The explicit form of the electron–phonon collision integral $I^{el-ph}(n_\epsilon, N^0_{\omega_q})$ and $Q(n_{\pm\epsilon})$ are given in Refs. [9, 12]. The dissipative steady state solutions of Equation (5) give the electron and the hole distribution functions in the thin film electrode. The corresponding phonon fluxes emitted to (or absorbed from) its environment in a frequency interval $d\omega_q$ centered at frequency ω_q can be obtained from (4).

The number of phonons absorbed per time unit in the volume Ω at frequency ω_q in the spectral interval $\mathrm{d}\omega_q$ is given by

$$\mathrm{d}\dot{N}_{\omega_q} = -L\left(N^0_{\omega_q}\right)\rho_0(\omega_q)\mathrm{d}\omega_q = \rho_0(\omega_q)I^{ph-el}\left(N^0_{\omega_q}\right)\mathrm{d}\omega_q \,. \tag{6}$$

Here $\rho_0(\omega_q) = \Omega\omega_q^2/(2\pi^2u^3)$, u is the velocity of sound and $\dot{X} \equiv \partial X/\partial t$. The absorbed power per volume Ω at frequency ω_q in the spectral interval $\mathrm{d}\omega_q$ is given by

$$\frac{\mathrm{d}P}{\mathrm{d}\omega_q} = \hbar\omega_q\ \rho_0(\omega_q)\ I^{ph-el}(N^0_{\omega_q})\ . \tag{7}$$

Introducing the operator $I(N^0_{\omega_q})$ via

$$I(N^0_{\omega_q}) = \frac{16\pi\epsilon_F}{\lambda\omega_D}\,\omega_q^3 I^{ph-el}\left(N^0_{\omega_q}\right),$$

allows to express the absorbed power (7) in dimensionless form

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\Omega\lambda\omega_D}{16\pi\epsilon_F u^3}\frac{(k_BT_{c'})^5}{\hbar^3}I(N^0_x) = P_0\ I(N^0_x)$$
$$P_0 = \frac{\Omega\lambda\omega_D}{16\pi\epsilon_F u^3}\frac{(k_BT_{c'})^5}{\hbar^3}\ , \tag{8}$$

where we abbreviate $x = \hbar\omega_q/(k_BT_{c'})$ and $T_{c'}$ denotes the critical temperature of the injector.

In the following we use the units $k_B = \hbar = e = 1$ if not stated otherwise. Also, recall that the primed variables correspond to the injector material.

A. Numerical results and analysis

To simplify further the analysis, we need to analyze properties of equation (5). The electron–phonon collision integral I^{el-ph} is proportional to the parameter $\gamma \propto T^3/\omega_D^2$ (damping) which characterizes the strength of the relaxation processes. The parameter ν describes the intensity of the particle and hole injection $Q(n_{\pm\epsilon})$. The typical values of ν are small compared to γ. When ν/γ tends to zero, the particle and hole distribution functions go over to thermal equilibrium distribution functions and both, $I^{el-ph}(n_{\pm\epsilon}, N^0_{\omega_q})$ and $I^{ph-el}(N^0_{\omega_q}, n_{\pm\epsilon})$ vanish. At finite values of ν/γ the particle and hole distribution functions differ from the thermal equilibrium functions and thus phonon fluxes arise. As observed in Refs. [8, 9, 18] both negative and positive phonon fluxes occur between the thin film and the thermal reservoir. The net phonon flux integrated over all frequencies can be either negative or positive.

Figure [2] shows the power $P(V)/P(0)$ absorbed by the normal metal thin film electrode. It is shown as a function of the potential applied across the $S'IN$ tunnel junction for different values of the parameter ν/γ. All curves have a minimum (maximal absorption) at $V < \Delta'(T)$. When the potential difference V across the tunnel junction approaches the superconducting gap $\Delta'(T)$, the phonon absorption decreases. The absorbed power increases with the increasing values of ν/γ. The situation changes dramatically in the high temperature regime $T/T_{c'} > 0.5$. When the value of the potential applied across the tunnel junction approaches the superconducting gap Δ', then the $S'IN$ tunnel junction starts to emit phonons to the environment. This is shown in Fig.[2b]. In Fig. [3] we present the dynamics of phonon flux in the vicinity of the minima of Figs [2a,b]. As expected from Fig.[2a], the phonon absorption spectrum is spread out over the whole frequency range of acoustic phonons at the temperature $T/T_{c'} = 0.5$ and $\Delta' - V > 0.18$. At temperature $T/T'_c = 0.8$ there is both phonon emission and phonon absorption at high and low frequency regions, respectively. The phonon emission spectrum of a SIN tunnel junction is different from the $S'IS$ case (see Fig.[1]). The SIS tunnel junction has two emission and one absorption frequency regions. In SIN tunnel junctions the low frequency emission region is absent. Also, as temperature decreases the phonon emission decreases faster than the absorption.

B. PDE in the SIN tunnel junctions:"Equilibrium" limit

An analytic expression for the power absorbed by the SIN tunnel junction when electron and phonon subsystems are maintained at temperature T_e and T_{ph}, respectively, can be obtained by integrating expression (8). The heatflow from the phonon subsystem to the electron subsystem maintained at different temperatures is given then by (for details see Appendix)

$$P = \alpha\left[\left(\frac{T_e}{T_{c'}}\right)^5 - \left(\frac{T_{ph}}{T_{c'}}\right)^5\right], \tag{9}$$

where $\alpha = 4P_0\ \Gamma(5)\ \zeta(5)$, $\zeta(x)$ denotes Riemann's Zeta function and $\Gamma(n)$ is the Gamma function. A comparison of Equation (9) with the Equation (19) of Ref. [1] shows that $P \propto T_e^5 - T_{ph}^5$ is compatible with the PDE in SIN tunnel junctions [see Refs. [16, 23]]. This result shows that the usual approach [2, 6, 15, 16] takes into account only the net phonon flux. In this approach details like absorption at low frequency region and emission at high frequency region (see Fig. 3) are inaccessible. However, the details of phonon absorption spectra are important and may open new possibilities for cooling by SIN tunnel junctions as discussed in the next Section.

IV. MICROREFRIGERATOR BASED ON THE PHONON DEFICIT EFFECT

Why is it important to know the spectral dependence of the phonon fluxes? How can the phonon-deficit-effect

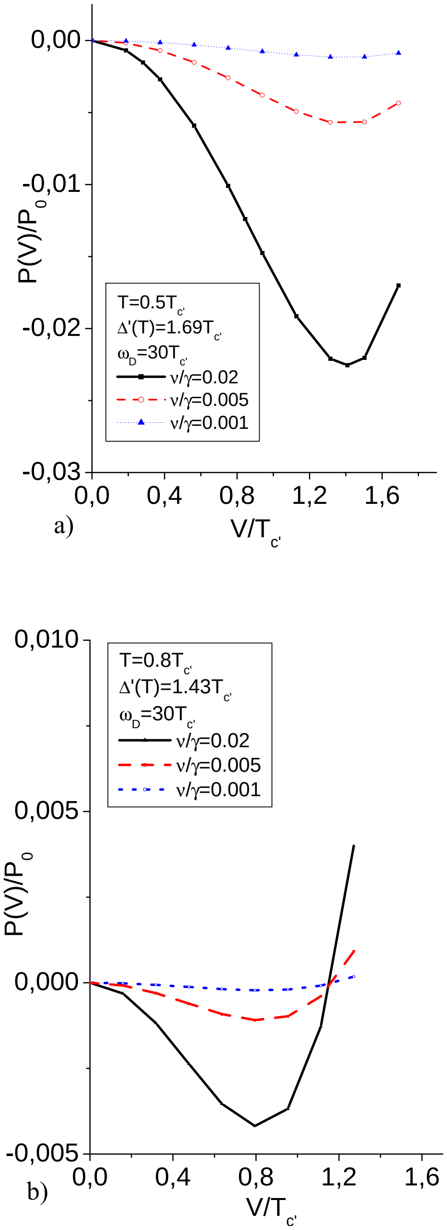

FIG. 2: Absorbed power $P(V)$, in units of P_0, by the normal metal electrode of the thin film as function of applied potential for different values of ν/γ. $\Delta'(T)$ and $T_{c'}$ are the superconducting gap and the critical temperature of the injector, respectively, ω_D is the Debye frequency of the normal metal electrode of the thin film and V is the applied potential across the tunnel junction. (a) $T/T_{c'} = 0.5$, (b) $T/T_{c'} = 0.8$. The lines are a guide to the eye.

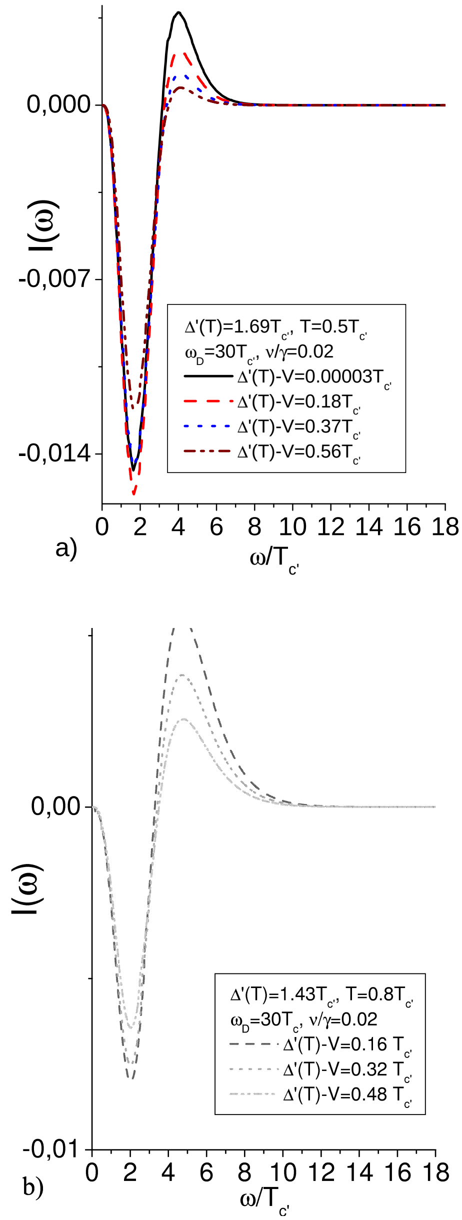

FIG. 3: Spectral absorption (negative part) and emission (positive part) of phonons near the maximal phonon absorption potential of the thin film. $\Delta'(T)$ and $T_{c'}$ are the superconducting gap and the critical temperature of the injector, respectively. ω_D is the Debye frequency of the normal metal electrode of the trhin film and V is the applied potential across the tunnel junction. (a) $T/T_{c'} = 0.5$, (b) $T/T_{c'} = 0.8$.

influence the microrefrigerator design? The answers to these questions can be found in *spectral-selective* phonon filters.

To clarify this point, we should calculate the energy, which the nonequilibrium phonons yield to the heatbath. This quantity is equal to the integral of the spectral function plotted in Fig.[1]. This integral having a net negative value is likely to be higher in the case of an asymmetric $S'IS$ junction [8, 18] and also in asymmetric $S'IN$ junctions. Fig.[1] displays the nonequilibrium phonon flux from the $S'IS$ tunnel junction, $\Delta_S << \Delta_{S'}$. As follows from Figs.[1], [2] and [3] the net flux integrated over energy is negative [7, 8, 18]. To enhance the PDE effect and achieve better cooling, one should separate

nonequilibrium negative and positive fluxes across the interface between the superconductor and the attached heat reservoir. Let us consider placing a phonon spectral filter between superconductor S and reservoir 2 (Fig.[4]). Suppose we can construct a filter [13, 17] which is transparent to phonons in a window of frequencies where the flux in Fig.[1] is negative. Such a filter would permit the flow of phonons ($\hbar\omega_{\mathbf{q}} \approx 2\Delta$) from the reservoir 2 to the superconductor S. At the same time the filter will absorb phonons below and above that frequency window, including those coming from the heated barrier. Another filter with complementary (i.e. the band pass) transmission properties may be placed between reservoir 1 and the electrode S'. It will prevent phonons coming from S' (i.e., the reservoir 1) from being absorbed in S. This will make the absorption from the reservoir 2 more efficient. Then the system shown in Fig.[1] will work as a refrigerator, by coolingthe reservoir 2. This cooling effect cannot reduce the temperature much below the critical temperature of superconductor S' (Fig.[1]). However, there are different classes of superconductors with critical temperatures covering a wide range of temperatures from about $150K$ to very low temperatures. Then, cooling cascades can be organized in such a way to generate substantial cooling. We suggest that such designs of a PDE-based microrefrigerator could be carefully considered for the purpose of practical use. In the next section we will discuss some designs.

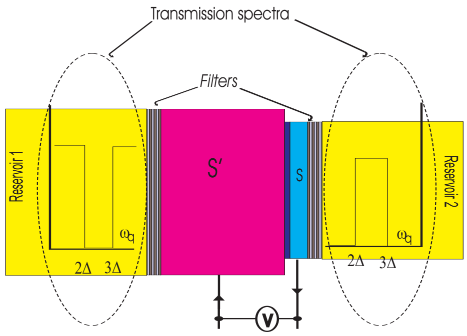

FIG. 4: Layout of the microrefrigerator with the improved properties. Inserts show suggested spectral characteristics of superlattice filters.

A. Design of phonon filters

From the analysis in the preceding subsection it follows that the design of appropriate phonon filters is of primary importance. Using the analogy between reflection of a plane electromagnetic wave at the interface between two optical media with different refractive indexes n_1 and n_2 and the acoustic wave reflecting at the boundary of two elastic media with different acoustic impendance Z_1 and Z_2, Narayanamurti et al. [14, 19] have developed appropriate phonon filters for phonon spectrometery.

Here we consider the simplest case of a phonon filter [17, 18] for the case when acoustic phonons propagate through a superlattice perpendicular to interfaces. In Fig.[5] the superlattice system is shown. The system

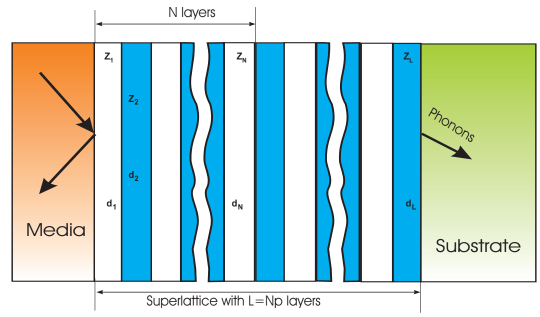

FIG. 5: Bragg multilayer between two media.

consists of a sequence of different layers: material 1 with thickness d_1 and impedance Z_1, material 2 with thickness d_2 and impedance Z_2, etc. If one repeats this configuration p times then the superlattice will have a period $D = d_1 + d_2 + \ldots + d_n$ and $L = Np$ layers. The interfaces are perpendicular to the wave vector of the incident wave. The superlattice is placed between two media with the acoustic impedance Z_{medium} and $Z_{substrate}$. Methods of calculation for the transmission and reflection coefficients in a linear stratified medium are described in Refs. [3, 20, 21, 22]. We have presented results of our calculation in Fig.[6]. One should note that all filters

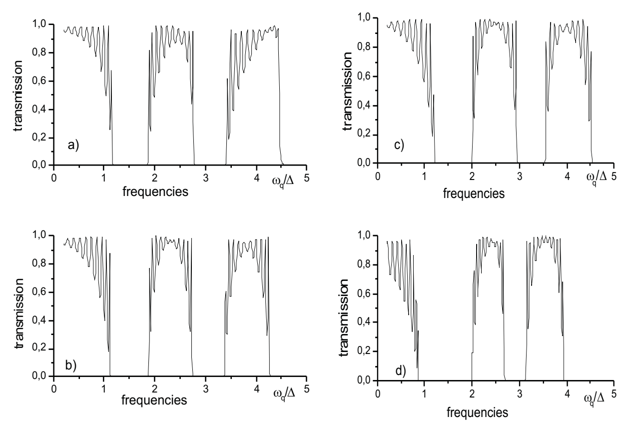

FIG. 6: Band pass properties of designed filters (for the parameters see Tab.[I]).

have an oscillatory behavior for the transmission coefficient in the band pass region. This is typical for band pass filters. In Tab.[I] we list impedance and thickness of the superlattices used in our calculations. Fig.[6] presents curves for the transmission spectra of the filter between

reservoir 2 and superconductor S with different acoustic impedance, layer thickness and layer number. Comparing with Fig.[1] we observe large phonon fluxes at frequencies about $\omega_{bfq} = \Delta/\hbar$ and $\omega_{bfq} = 3\Delta/\hbar$. The filter shown in Fig.[6d] may work well for those frequencies. The filter between reservoir 1 and superconductor S' is presented in Ref [17]. The phonon filters for the SIN tunnel junctions can be designed by a similar analysis.

TABLE I: Parameters of the presented filters (see Figure 6). The impedance (Z) and the size of the layers (d) are given in units of $10^5 g/(s\ cm^2)$ and in Angstroms (Å), respectively.

	Z_1	Z_2	Z_3	Z_4	Z_5	d_1	d_2	d_3	d_4	d_5	N	p
a	2.5	1.8	1.4	0.9	1.8	50	50	50	50	30	5	11
b	3	1.8	1.3	0.9	1	50	50	50	50	30	5	11
c	3	1.8	1.3	0.9	1	40	45	50	50	30	5	11
d	3	1.8	1.3	0.9	1	40	40	40	45	30	5	11

B. Phonon filters and refrigeration

The proposed PDE refrigerator is suited to refrigerate an object attached to it. The effect of phonon filters can be estimated by the value of P/P_0 from an experiment carried out by Pekola et al [16] where a membrane attached to the SIN tunnel junction was cooled. Our estimate for the net phonon absorption P/P_0 by Equation (9) varies from 0 to 10^{-2} depending on temperatures T_e and T_{ph}. Table II presents our calculated values for the nonequilibrium tunnel junctions (see also Figure 3). We find qualitative agreement between experiment and calculation for the value of absorbed power.

TABLE II: Maximal absorbed power P/P_0 in the SIN tunnel junction at three different temperatures $T = 0.27T_{c'}, T = 0.5T_{c'}$ and $T = 0.8T_{c'}$.

$T/T_{c'}$	0.27		0.5		0.8	
$V/T_{c'}$	1.56		1.31		0.79	
ν/γ	0.005	0.02	0.005	0.02	0.005	0.02
P/P_0	0.004	0.014	0.0052	0.0224	0.0011	0.0042

The phonon filters will increase the absorbed power by cutting off the emission window (see Figure 3). Table III presents the absorbed power after placing a low pass filter (see Figures 3 and 7) between the SIN tunnel junction and an object. Phonon absorption is increased at $T/T_c = 0.5$. At $T/T_c = 0.8$ we see that SIN tunnel junction with phonon filter will absorb phonons (the minus sign in Table III means absorption). Fig. [6] shows the temperature dependence of the total phonon absorption P/P_0 for three different casses. The solid line is the phonon absorption by a SIN tunnel junction without phonon filter. The dotted and dashed curves present the filtered phonon absorption for two different low pass filters with transmittance 1 at $\omega < 2.95T_c$ and $\omega < 3.25T_c$. Thus the phonon absorption is incresed in a wide temperature region.

TABLE III: Absorbed power P/P_0 in the SIN tunnel junction at two different temperatures $T = 0.5T_{c'}$ and $T = 0.8T_{c'}$, without and with phonon filter. The data below correspond to Fig. [3]. The ratio $\nu/\gamma = 0.02$.

$T/T_{c'}$	0.5		0.8	
$V/T_{c'}$	1.31		1.31	
effect	total	filtered	total	filtered
P/P_0	$-2.24\,10^{-2}$	$-2.43\,10^{-2}$	$1.24\,10^{-4}$	$-2.5\,10^{-2}$

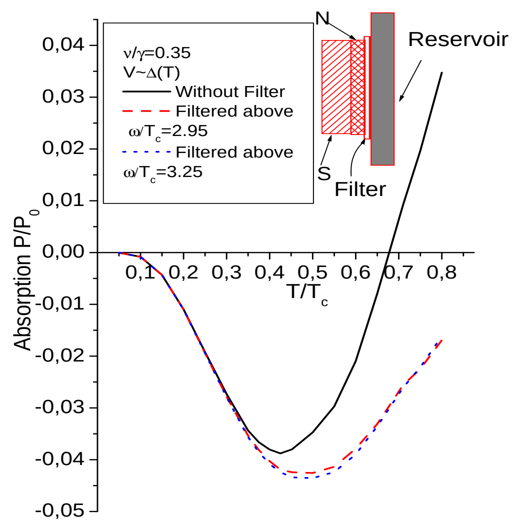

FIG. 7: Phonon flux with low pass filter at $\omega = 2.95T_c$ and $\omega = 3.25T_c$

We recall that the refrigeration schema presented above is suited to refrigerate an object attached to it. But as follows from the phonon absorption spectra an analogous design with high pass filters can prevent the phonon absorption by SIN tunnel junction and thus reduce the heating of electrons by phonons (see Figures [3, 7]).

V. CONCLUSIONS

The $SSMR$ performance depends on various external and internal parameters. The PDE-based approach presented here and the usual apprach [1, 16, 23] give the similar expression (Eq. 9) for the net cooling in case of SIN tunnel junction. Thus one can conclude that the cooling of a membrane in the experiment by Pekola et al. [16] is a direct implication of the phonon deficit effect.

The $PDE-$approach offers explicit spectral description for participating phonon fluxes. It provides receipes to overcome unwanted heating channels and shows new details which have been omitted in a simplified "cooling by extraction" description.

We have analyzed the "phonon deficit effect" in SIN tunnel junctions. In contrast to SIS tunnel junctions there is a temperature region where the SIN tunnel junctions essentially absorb low frequency phonons. As temperature is increased, the SIN tunnel junctions also emit phonons to the environment. Depending on the value of the ratio ν/γ there is net phonon emission or absorption. The temperature dependence of the net phonon absorption is consistent with experimental observations [6, 15, 16]: there is heating at high and low temperatures and refrigeration at intermediate temperatures.

To enhance the cooling power of the $SIS(N)$ and tunnel junctions and overcome the phonon heating at high temperatures, phonon filters can be effective. We performed calculations for the microrefrigerator phonon filters. Those filters have superlattice design and can be obtained using contemporary nanotechnology methods. They can enhance the cooling power of SIS and SIN tunnel junctions.

Acknowledgments

H.K. has been supported by NSERC Canada. G.M. is grateful for support by the "Programme québécois de bourses d'excellance" of the governement of Québec.

APPENDIX: PHONON FLUX BETWEEN ELECTRONS AND PHONONS

In this appendix, we give the derivation of the expression (9).

The general expression for the phonon flux is given by (compare with the formula (7), $\hbar = 1, k_B = 1$)

$$P = \int_0^{\omega_D} \omega_q \; \rho_0(\omega_q) \; I^{ph-el}(N^0_{\omega_q}) \mathrm{d}\omega_q. \tag{A.1}$$

Here $I^{ph-el}(N^0_{\omega_q})$ is the phonon-electron coillision integral and it is given by

$$\begin{aligned} I^{ph-el}\left(N_{\omega_q}\right) \;&=\; \frac{\pi\lambda\omega_D}{8\epsilon_F} \int_0^\infty \mathrm{d}\epsilon \int_0^\infty \mathrm{d}\epsilon' \left\{\mathcal{S}_1 \delta\left(\epsilon + \epsilon' - \omega_q\right)\right. \\ &\left. + \, 2\mathcal{S}_2 \delta\left(\epsilon - \epsilon' - \omega_q\right)\right\}, \end{aligned} \tag{A.2}$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ are elementary collison processes. When the distribution function of electrons and holes is Fermi function then $n_\epsilon = n_{-\epsilon}$. In this case $\mathcal{S}_1$ and $\mathcal{S}_2$ can be written as

$$\mathcal{S}_1(\epsilon, \epsilon', \omega_q) = 4\left[\left(N_{\omega_q} + 1\right) n_\epsilon n_{\epsilon'} - N_{\omega_q}\left(1 - n_\epsilon\right)\left(1 - n_{\epsilon'}\right)\right], \tag{A.3}$$

$$\mathcal{S}_2(\epsilon, \epsilon', \omega_q) = 4\left[\left(N_{\omega_q} + 1\right) n_\epsilon \left(1 - n_{\epsilon'}\right) - N_{\omega_q}\left(1 - n_\epsilon\right) n_{\epsilon'}\right]. \tag{A.4}$$

After substitutions of the expressions (A.3), (A.4) and (A.2) in the expression (A.1), the integration by delta function gives

$$\begin{aligned} P \;&=\; \frac{4\pi\lambda\omega_D}{8\epsilon_F} \int_0^{\omega_D} \mathrm{d}\omega_q \omega_q \; \rho_0(\omega_q) \left[\int_0^{\omega_q} \; \mathcal{S}_1(\epsilon, \omega_q - \epsilon, \omega_q) \; \mathrm{d}\epsilon \right. \\ &\left. + \int_{\omega_q}^{\omega_D + \omega_q} \; 2\mathcal{S}_2(\epsilon, \epsilon - \omega_q, \omega_q) \; \mathrm{d}\epsilon \right]. \end{aligned} \tag{A.5}$$

Finally evaluating these integrals by noting that $T \ll \omega_D$ and using the integral representation of Rieman zeta function $\zeta(n+1)\Gamma(n+1) = \int_0^\infty x^n/(exp(x) - 1)\, d\,x$ one obtains

$$P = 4\frac{\Omega}{16\pi} \frac{\lambda\omega_D}{\epsilon_F u^3} \Gamma(5)\zeta(5)\left(T_e^5 - T_{ph}^5\right). \tag{A.6}$$

[1] P.B. Allen. Theory of thermal relaxation of electrons in metals. *Phys. Rev. Lett.*, 59:1460, 1987.

[2] M.G. Castelano, R. Leoni, G. Torrioli, P. Carelli, and A. Gerardino. Characteristics of superconductor–insulator–normal tunnel junctions for on–chip electronic refrigeration. *Il Nuovo Cimento D*, 19:1417, 1997. paper presented at the VIII Congresso SATT, Como, 1-4 October 1996.

[3] Shun Lien Chung. Efficient band-structure calculations of strained quantum wells. *Phys. Rev. B*, 43:9649, 1991.

[4] G.M. Eliashberg. Film superconductivity stimulated by a high-frequancy field. *JETP Lett.*, 11:114, 1970.

[5] G.M. Eliashberg. Inelastic electron collisions and nonequilibrium stationary states in superconductors. *Sov. Phys. JETP*, 34:668, 1972.

[6] P.A. Fisher, J.N.Ullom, and M. Nahum. High-power on-chip microrefrigerator based on a normal-metal/insulator/superconductor tunnel junction. *Appl. Phys. Lett.*, 74:2705, 1999.

[7] A.M. Gulian, G.F. Zharkov, and G.M. Sergoyan. *Sov. Phys. Lebedev Inst. Reports*, (10):38, 1985.

[8] A.M. Gulian, G.F. Zharkov, and G.M. Sergoyan. Thermal balance in the tunnel contact of a superconductor. *Kratkie Soobshcheniya po Fizike FIAN*, (10):33, 1985.

[9] A.M. Gulian and G.F. Zharkov. The "phonon deficit" effect in superconductors induced by UHF radiation. *Phys. Lett. A*, 80:79, 1980.

[10] A.M. Gulian and G.F. Zharkov. Dependence of a superconducting gap on temperature in an UHF field. *JETP Lett.*, 34:153, 1981.

[11] A.M. Gulian and G.F. Zharkov. Electron and phonon kinetics in a nonequilibrium Josephson junction. *Zh. Eksp. i Teor. Fiz.*, 89:156, 1985.

[12] A.M. Gulian and G.F. Zharkov. *Electrons and Phonons in Nonequilibrium Superconductors*. Kluwer/Plenum, 1999.

[13] A.M. Gulian. Microrefrigeration and phonon deficit effect. *Physica B*, 263–264:621, 1999.

[14] O. Koblinger, J. Mebert, S. Döttinger E. Dittrich, W. Eisenmenger, P.V. Santos, and L. Ley. Phonon stop bands in amorphous superlattices. *Phys. Rev. B*, 35:9372, 1987.

[15] M.M. Leivo, J.P. Pekola, and D.V. Averin. Efficient Peltier refrigeration by a pair of normal metal/insulator/superconductor junctions. *Appl. Phys. Lett.*, 68:1996, 1996.

[16] A.J. Manninen, M.M. Leivo, and J.P. Pekola. Refrigeration of a dielectric membrane by superconductor/insulator/normal-metal/insulator/superconductor tunneling. *Appl. Phys. Lett.*, 70:1885, 1997.

[17] G.G. Melkonyan, H. Kröger, and A.M. Gulian. Application of bragg superlattice filters in low temperature microrefrigerators. In Roger Lessard and George A. Lampuopoulos, editors, *International Conference of the Applied Photonic Technologies (ICAPT), Quebec. Proceedings of SPIE*, volume 4087, page 525, Québec, 2000.

[18] G.G. Melkonyan, H. Kröger, and A.M. Gulian. Nonequilibrium superconducting tunnel junction and the phonon deficit effect. *Physica B*, 284–288:2032, 2000.

[19] V. Narayanamurti, H.L. Störmer, M.A. Chin, A.C. Gossard, and W. Wiegmann. Selective transmission of high-frequency phonons by a superlattice: The "dielectric" phonon filter. *Phys. Rev. Lett.*, 43:2012, 1979.

[20] D. Pierce. *Acoustics: An Introduction to its Physical Principles and Applications*. McGraw–Hill, 1981.

[21] P.I. Richards. Application of matrix algebra to filter theory. In *Proceedings of the I.R.E*, volume 34, page 145, 1946.

[22] S.A. Shelkunoff. The impedance concept and its application to problems of reflection, refraction, shielding and power absorption. *Bell Sys. Tech. J.*, 17:17, 1938.

[23] F.C. Wellstood and C. Urbina amd John Clarke. Hot-electron effects in metals. *Phys. Rev. B*, 49:5942, 1994.